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THE ORBITAL GEOMETRY OF JUPITER'S MOONS*

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Qualitative geometrical and differential-geometrical characteristics of the orbits of the moons of Jupiter in its gravitational field are studied (up to the fourth zonal harmonic).

Eliminating the cylical coordinate from the energy integral, we determine the domains of possible motions of the moons. The boundaries of these domains are given in the ρz plane by the Hill curves

$$W(\rho, z) + h = 0 \quad (W = U - C^2/(2\rho^2)) \tag{1}$$

Here W is the modified force function, ρ and z are the cylindrical coordinates of the moon, U is the gravitational potential of Jupiter /1/ and C is the area constant. The Hill boundary which is usually considered, derived from the energy integral in its initial form, describes approximately the domain of possible motions in the form of a spheroid, while in the present case the domain in question will be a spheroidal layer.

Let us replace the force function in (1) by its expression /2, 3/

$$U = \frac{fM}{r}\left[1 + \sum_{k=2}^{\infty} I_k \left(\frac{R}{r}\right)^k P_k(\sin\varphi)\right], \quad \sin\varphi = \frac{z}{r} \tag{2}$$

in which f is the gravitational constant, M is the mass of Jupiter, R is the mean equatorial radius, r is the planetocentric distance of the moon, φ is the planetocentric latitude, I_k are dimensionless constants, and $P_k(\sin\varphi)$ is a k-th order Legendre polynomial. When $I_k = 0$, the equation of the Hill curve takes the following form:

$$r_0 = a\,(1 \pm \sqrt{1 - \cos^2 i \sec^2 \varphi}, \quad \cos i = C/(fMa)^{1/2}$$

where a and i denote the major semi-axis and the inclination of the Keplerian orbit. In the case when $I_k \neq 0$, we assign appropriate values to the constants I_k to obtain the domains of possible motions, which can be used to assess the effect of the asphericity of Jupiter on the orbits of the moons.

Retaining in the gravitational potential of Jupiter only the second and fourth zonal harmonics, we shall write the equation of the Hill curve (1) in the form (l is the eccentricity of the orbit)

$$r = r_0 - r_1 I_2 + r_2 I_2^2 + R^4 \frac{5 + \eta\,[8 + 5(7\eta + 5\eta^2)]}{64a^3 S(\xi, \eta)} \times I_4 \tag{3}$$

$$r_1 = \frac{R^2(2 - \eta - 3\eta^2)}{8aS(\xi, \eta)}, \quad r_2 = \frac{R^2 r_1^2 (2 + 2b(1 + \eta)) - 3(1 - e^2)(1 + \xi)}{r_0 S(\xi, \eta)}$$

$$c^2 = {}^1/_2 fMa\,(1 - e^2)(1 + \xi), \quad b = r_0/a, \quad S(\xi, \eta) = {}^1/_2\,[(1 - e^2)(1 + \xi) - b(1 + \eta)]$$

$$\eta = \cos 2\varphi, \quad \xi = \cos 2i$$

The geometrical characteristics of the motion of Jupiter's moons were studied using the values of the astronomical constants given in /1, 3/. Figure 1 shows the domains of possible motions of Jupiter's moons and Fig.2 shows the perturbations in the radius vector of the boundary Hill curve caused by the asphericity of Jupiter (the notation is the same as in Fig. 1). Figure 1 shows that the Hill curves are ovals. Since some of them intersect each other collisions between the moons cannot be ruled out.

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Thus the qualitative analysis shows that when the altitude of the moon is increased while the orbit eccentricity e and angle of inclination i are reduced, the Hill curves contract and become similar to a circle.

In order to study a bundle of trajectories emerging from a single point, we shall consider a Darwin curve defined as the geometrical position of the singularities

$$W_\rho'^2 = W_z'^2 \tag{4}$$

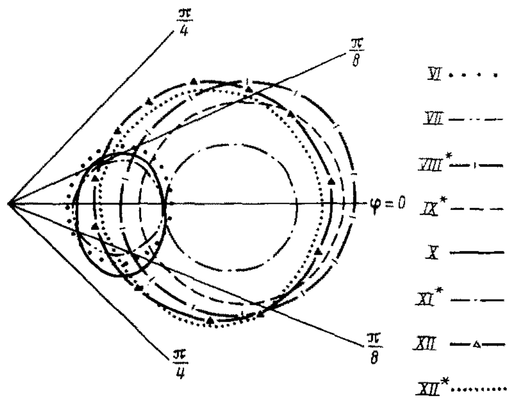

Fig.1

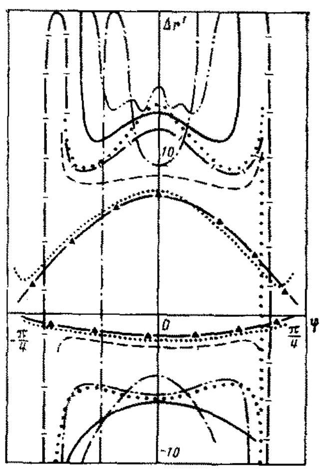

Fig.2

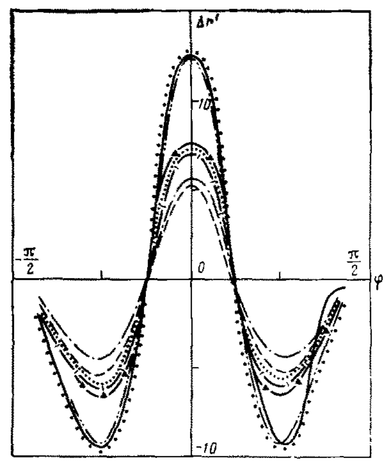

Fig.3

Curve (4) intersects the Hill curve (3) only at the singularities, and even then only when the values of the constant h_* correspond to these singularities. We have, for any value of $h > h_*$, a certain region near each singularity at every point of which the Darwin characteristic $D^\circ(h) = W_\rho'^2 - W_z'^2 < 0$ and hence the bundle of trajectories, will be elliptical.

Let us consider the equation of the characteristic of the contacts between the trajectories, and a one-parameter family of circles

$$q(\rho, z) = \rho^2 + z^2 = d^2 \tag{5}$$

Taking into account the energy integral we can obtain, for $q^{\cdot\cdot\circ}(\rho, z)$,

$$q^{\cdot\cdot\circ}(\rho, z) = 2W + 2h + \rho W_\rho' + z W_z' = 0 \tag{6}$$

Replacing the force function by expression (2) in (6), we obtain the equation for the characteristic of the contacts

$$\begin{gathered} q^{\cdot\cdot}(r) = r_0 + r_1 I_2 + r_2 I_2^2 + \frac{105 R^4}{32 r_0^3}(1 - 4\eta + 3\eta^2) I_4 \\ r_1 = \frac{R^2}{4a}(8 - 9\eta), \quad r_2 = \frac{R^4}{8 r_0^3}(5 + 18\eta + \eta^2) \end{gathered} \tag{7}$$

Curve (7) separates the region of internal contacts from the region of external contacts, i.e. we can use it to separate the region of pericentres from the region of apocentres.

Let us consider the differential-geometrical Hadamard characteristic (the direction of the concavity of the trajectories). We will use it to determine the inner region bounded by the curve (7). The contact within this region will be external for any possible trajectory.

The Hadamard curve is given by the equation

$$H^{\circ}(r) = r_0 + \frac{1}{b(1+\eta) + 2(1-e^2)(1+\xi)} \times \tag{8}$$
$$\left[r_1 I_2 + r_2 I_2{}^2 - \frac{15R^3}{32ar^2}(6 + 21\eta - 7\eta^2 + 21\eta^3) \right] I_4$$
$$r_1 = \frac{3R^2}{4a}(1-\eta)\,[4 - (5-2\zeta)\,\eta], \quad r_2 = \frac{r_1{}^2}{r_0}\{12ar_1(1-e^2)(1-\xi) -$$
$$(4r_1r_0 - 3R^2(1-\eta))\,[4 - (1-\zeta)\,\eta]\}, \quad \zeta = \sin\varphi$$

In order to determine the position of curve (8) more accurately, we shall consider the distributrix /4/

$$K(\rho, z) = \rho W_\rho{}' + zW_z{}' = 0 \tag{9}$$

The singularities of curve (9) have a common tangent to the level lines. It is clear that the Hadamard curve lies wholly within this curve, and has the form of a lemniscate, has a discontinuity at $\varphi = 90°$, a maximum at $\varphi = 0$ and a minimum at $\varphi = \pm 50°$. When the altitude and eccentricity of the moon are both increased (decreased), the Hadamard curve will move away from (towards) the origin of coordinates (Fig.3, the notation used is the same as in Fig.1).

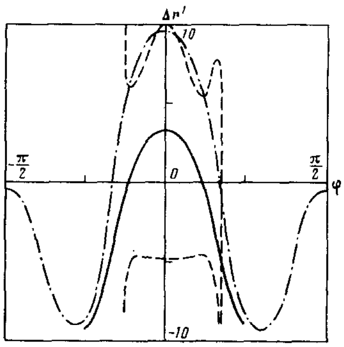

Fig.4

Figure 4 shows the displacements of the Hill curve (dashed line), the Hadamard curve (dash-dot line) and the contact curve (solid line) for moon (VI) of Jupiter.

Calculations show that in the case of the Amalti (V), Ganimede (III) and Callisto (V) moons the influence of the second and fourth zonal harmonic is substantial. For example, at the pericentre the boundaries of the possible motion show the following deviations at the minimum point: 302m (V), 508m (III) and 593m (IV).

For moon X the deviation is 0.2m and for the remaining moons it is less than 0.01m.

At the apocentre the boundary of the region of possible motions has the following deviations at the minimum point: 0.3m for XI, 0.02m for III, IV, V and (VI), and less than 0.01m for the remaining moons.

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